Mathematical model and optimization of solder microstructure in a three-layer beam

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Thermal stresses that occur during soldering, vary within wide limits depending on the nature of the temperature fields, the geometric configuration and material properties. Substantial levels of stress can lead to cracking, plastic deformation and other undesirable phenomena which reduce the strength of the solder joint. Optimization of the solder layer topology can solve the problem of excessive stresses within the specified design and technological limitations. The report presents a mathematical formulation and method of solving a wide class of optimization problems that arise in connection with the heating of multilayer plates.

Consider the element of the unit length of the solder joint of the beam (Fig. 1) made of two layers with different materials, which are soldered together by a thin layer of solder. Assume the width of the beam $2l$, $l=2{,}5cm$. The table shows the thickness, Young modules, Poisson coefficients and coefficients of thermal expansion of layers h_i, E_i, ν_i, α_i, respectively, ($i=1$ – the upper layer of iron-Nickel alloy (5Ni), $i=2$ – a layer of solder tin, $i=3$ – the lower layer of quartz glass). In the initial state the beam is at a melting point of solder ($\theta_{solder}=232$ °C) without stress.

	h_i, cm	E_i, N/cm^2	ν_i	α_i,1/ grad
$i=1$	1	$20.58\cdot10^6$	0.3	$6.7\cdot10^{-6}$
$i=2$	0.1	$5.39\cdot10^6$	0.3	$15\cdot10^{-6}$
$i=3$	1	$6.86\cdot10^6$	0.3	$2\cdot10^{-7}$

When the beam is cooled down after soldering to the operating temperature θ_{exp}, displacements u_i, deformations ε_{ij} and stresses σ_{ij} occur in the elastic field. The behavior of the linear thermoelastic body is determined by the defining relations for the stress tensor σ, of strain tensor ε and deviation θ of the operating temperature $\theta_{exp}=25$ °C from the soldering temperature θ_{solder}.

$$\sigma_{ij} = 2G\left\{\varepsilon_{ij} + \frac{1}{1-2\nu}\left[\nu\varepsilon_{kk} - \alpha(1+\nu)\theta\right]\delta_{ij}\right\}. \quad (1)$$

The thermoelasticity problem is described by a system of differential equations:

$$\sigma_{ij,j} + \rho F_i = 0, \quad (2)$$

where ρ - density, G - shear modulus, E - modulus of elasticity, ν - Poisson's ratio, α - coefficient of linear thermal expansion. Equation (2) must be solved under the following boundary conditions: $\sigma n=0$.

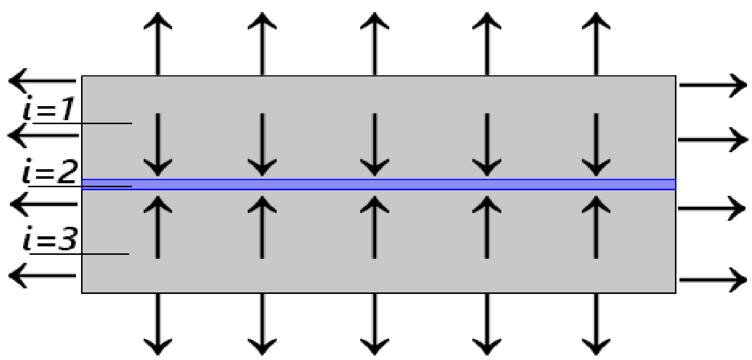

Figure 1. The element solder joint of beam.

Topology optimization the shape of the solder layer is in search of a better distribution of the greatest amount of solder on the layer 2 with a constraint on the magnitude of shear stress σ_{12}. In the topological optimization method [1-3], the Young modulus in the optimizable domain Ω, in our case it is a solder layer 2, is a function of artificially introduced material density $\rho(x)$. The stress tensor is considered to be a function of the Young module E_2 – of the solder material and $\rho(x)$ is a control variable in the optimization problem:

$$E(\mathbf{x}) = \rho(\mathbf{x})^p E_2, \quad \mathbf{x} \in \Omega.$$

With $\rho(x)=1$, the entire area is completely filled with material. Thus, you must find $\max\limits_{\rho(\mathbf{x})} \int\limits_{\Omega} \rho(\mathbf{x}) d\Omega$, with the constraint $\int\limits_{\Omega} \sigma_{12} d\Omega \le \int\limits_{\Omega} \sigma_{12}^{\max} d\Omega$, where $\sigma_{12}^{\max}$ - the set limit on the level of shear stresses. Problem (2) is solved by finite element method. As a result, optimal distributions of solder material for two constraints (Fig. 2) were obtained.

(a)

(b)

Figure 2. Optimal distribution of the solder material.

When restricted to a shear stress of 1000 N (Fig. 2a) the amount of solder material amounted to 0.87, while limiting in 1145 N (Fig. 2b) the amount of material was 0.92. For the same amount of material with homogeneous distribution, the stresses were 1168.4 N and 1194.4 N respectively.

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